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WEIGHTED INTUITIONISTIC FUZZY SOFT SET BASED DECISION MAKING

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Abstract

Intuitionistic fuzzy soft set (IFSS) is an important generalization of fuzzy soft set (FSS). In this research paper, we introduced the notion of weighted intuitionistic fuzzy soft set (WIFSS) as a generalization of intuitionistic fuzzy soft set (IFSS), which makes the description of the objective world more realistic, practical and accurate in most cases, making it very promising in all respect. We defined the concept of level soft sets in a weighted intuitionistic fuzzy soft set context and presented an adjustable approach to WIFSS based decision making for solving decision-making problems in an imprecise situation with some practical and illustrative examples.

MSC 2010 Classification codes: 03E70, 03E75

Key words and phrases: fuzzy set, intuitionistic fuzzy set, intuitionistic fuzzy soft set, soft set, universe set, weight function and weighted intuitionistic fuzzy soft set.

Introduction

In real life, there are several complex challenges we encountered in environment, engineering, economics, medical sciences, social sciences, management sciences, etc. These kinds of problems are associated with uncertainties and imprecision in their domain. In search of solution to these problems, many mathematical tools were developed to deal with the problem of uncertainties and the imprecision. These theories among others include theory of probability (Prade and Dubois, 1980), theory of fuzzy sets (Zadeh, 1965), theory of interval mathematics (Atanassov, 1994), rough sets (Pawlak, 1982), vague sets (Gau and Buehrer, 1993) which we can consider as mathematical tools for dealing with uncertainties. But it was discovered that all these theories have their short comings in handling the level of uncertainties. The major setback associated with these theories is their incompatibility with the parameterization tools. To surmount these limitations, (Molodtsov, 1999) discovered or introduced the concept of soft set as a new mathematical tool for dealing with uncertainties and imprecision that is free from the difficulties that have troubled the standard existing mathematical models. Molodtsov pointed out the application of soft set in several directions. This theory has proven useful in many different fields such as decision making (Roy and Maji, 2007), data analysis (Zou and Xiao, 2008), forecasting and so on.

Research on soft sets has received considerable attention, since its introduction by Molodtsov in 1999 up to the present and several important results have been achieved in theory and practice. Maji *et al.* (2003) defined many algebraic operations in soft set theory and published a detailed theoretical study on soft sets. Ali *et al.* (2009) further presented and investigated some new algebraic operations for soft sets. Sezgin and Atagun (2011) proved that certain

De Morgan's law holds in soft set theory with respect to different operations on soft sets and discuss the basic properties of operations on soft sets such as intersection, extended intersection, restricted union and restricted difference. Thereafter, it was observed that soft set can be combined with other mathematical models. Maji *et al.* (2001) were the first to present fuzzy soft set. Maji *et al*. (2003) established classical soft set to intuitionistic fuzzy soft sets, which were further discussed in (Maji *et al*., 2003) and (Yin *et al*., 2012). Since then many researchers have established various hybrid soft sets.

Since decision-making is an integral part of our day-to-day activities, hence the study of decision making in an imprecise environment becomes imperative. Intuitionistic fuzzy soft set and its application in decision making has been studied by (Onyeozili and Balami, 2019). But we observed that not all the decision parameters are of equal importance to the decision maker(s), hence the need to impose weight to parameters. In this regard, the decision maker(s) imposes higher weight to parameters of interest.

As a generalization of intuitionistic fuzzy soft set theory, we introduced weighted intuitionistic fuzzy soft set theory that makes descriptions of the objective world more realistic, practical and accurate in some cases, making it very promising. In this paper, we use the approach introduced by (Jiang *et al.,* 2011) and construct some practical problems involving decision making. The definitions of the level soft sets are found in (Jiang *et al.,* 2011).

Preliminaries

Fuzzy Set

Definition 1 (Zadeh, 1965). Let U be a universe. A **fuzzy set** X over U is a set defined by a function μ_X representing a mapping,

$$\mu_X: U \longrightarrow [0,1]$$

μ_X is called the membership function of X, and the value $\mu_X(u)$ is called the grade of membership of $u \in U$ and represents the degree of u belonging to the fuzzy set X. Thus a fuzzy set X over U, can be represented as follows:

$X = \left\{\frac{u}{\mu_X(u)} : u \in U, \mu_X(u) \in [0,1]\right\}$ or $X = \left\{\frac{\mu_X(u)}{u} : u \in U, \mu_X(u) \in [0,1]\right\}$ or

$$X = \{\langle u, \mu_X(u) \rangle : u \in U, \mu_X(u) \in [0,1]\}.$$

Example 1. Let $U = \{h_1, h_2, h_3, h_4\}$. A fuzzy set X over U can be represented by

$$X = \left\{\frac{h_1}{0.4}, \frac{h_2}{0.6}, \frac{h_3}{0.2}, \frac{h_4}{0.7}\right\}$$

Intuitionistic Fuzzy Set

Definition 2 (Atanassov, 1986). Let X be a nonempty set. An **intuitionistic fuzzy set** A in X is an object having the form

$A = \{\langle x,\ \mu_A(x), \lambda_A(x)\rangle : x \in X\}$, where the functions $\mu_A(x), : X \longrightarrow [0,1]$ and

$\lambda_A(x): X \longrightarrow [0,1]$ defined respectively, the degree of membership and degree of non-membership of the element $x \in X$ to the set A, which is a subset of X and for every element $x \in X,\ 0 \le \mu_A(x) + \lambda_A(x) \le 1$.

Furthermore, we have $\pi_{\mathrm{A}}(x) = 1 - \mu_{\mathrm{A}}(x) - \lambda_{\mathrm{A}}(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_{\mathrm{A}}(x)$ is the degree of indeterminacy of $x \in X$ to the intuitionistic fuzzy set A and $\pi_{\mathrm{A}}(x) \in [0,1]$, that is, $\pi_{\mathrm{A}}(x): X \longrightarrow [0,1]$ and $0 \leq \pi_{\mathrm{A}} \leq 1$ for every $x \in X$. $\pi_{\mathrm{A}}(x)$ expresses the lack of knowledge of whether x belongs to intuitionistic fuzzy set A or not.

For instance, let A be an intuitionistic fuzzy set with $\mu_{\mathrm{A}}(x) = 0.55$ and $\lambda_{\mathrm{A}}(x) = 0.25, \Longrightarrow \pi_{\mathrm{A}}(x) = 1 - (0.55 + 0.25) = 0.2$. It can be interpreted as the degree that the object x belongs to intuitionistic fuzzy set A is 0.55, the degree that the object x does not belong to the intuitionistic fuzzy set A is 0.25 and the degree of hesitancy is 0.2.

Basic Operations on Intuitionistic Fuzzy Set

Let $A = \{\langle x, \mu_{\mathrm{A}}(x), \lambda_{\mathrm{A}}(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_{\mathrm{B}}(x), \lambda_{\mathrm{B}}(x)\rangle : x \in X\}$ be two intuitionistic fuzzy sets over X.

(i) [**Inclusion**] $A \subseteq B \leftrightarrow \mu_{\mathrm{A}}(x) \leq \mu_{\mathrm{B}}(x)$ and $\lambda_{\mathrm{A}}(x) \geq \lambda_{\mathrm{B}}(x), \forall x \in X$.
(ii) [**Equality**] $A = B \leftrightarrow \mu_{\mathrm{A}}(x) = \mu_{\mathrm{B}}(x)$ and $\lambda_{\mathrm{A}}(x) = \lambda_{\mathrm{B}}(x), \forall x \in X$.
(iii) **[Complement]** $A^C = \{\langle x, \lambda_{\mathrm{A}}(x), \mu_{\mathrm{A}}(x)\rangle : x \in X\}$.
(iv) **[Union]** $A \cup B = \{\langle x, max(\mu_{\mathrm{A}}(x), \mu_{\mathrm{B}}(x)), min(\lambda_{\mathrm{A}}(x), \lambda_{\mathrm{B}}(x))\rangle : x \in X\}$.
(v) **[Intersection]** $A \cap B = \{\langle x, min(\mu_{\mathrm{A}}(x), \mu_{\mathrm{B}}(x)), max(\lambda_{\mathrm{A}}(x), \lambda_{\mathrm{B}}(x))\rangle : x \in X\}$.

Soft Set

We first recall some basic notions in soft set theory. Let U be an initial universe set, E be a set of parameters or attributes with respect to U, $P(U)$ be the power set of U and $\mathrm{A} \subseteq \mathrm{E}$.

Definition 3 (Molodtsov, 1999). A pair (F, A) is called a **soft set** over U, where F is a mapping given by $F: A \rightarrow P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U. For $x \in A$, $F(x)$ may be considered as the set of x-elements or as the set of x-approximate elements of the soft set (F, A). The soft set (F, A) can be represented as a set of ordered pairs as follows: $(F, A) = \{(x, F(x)), x \in A, F(x) \in P(U)\}$

Definition 4 (Maji *et al.*, 2003). Let (F, A) and (G, B) be two soft sets over U. Then
(i) (F, A) is said to be a **soft subset** of (G, B), denoted by $(F, A) \tilde{\subseteq} (\mathrm{G}, \mathrm{B})$, if $A \subseteq \mathrm{B}$ and $F(x) \subseteq \mathrm{G}(x), \forall x \in \mathrm{A}$
(ii) (F, A) and (G, B) are said to be **soft equal**, denoted by $(F, A) = (\mathrm{G}, \mathrm{B})$, if $(F, A) \tilde{\subseteq} (\mathrm{G}, \mathrm{B})$ and $(\mathrm{G}, \mathrm{B}) \tilde{\subseteq} (F, A)$.

Definition 5 (Ali *et al.*, 2009). Let (F, A) be a soft set over U. Then the **support** of (F, A) written **supp**(F, A) is defined as **supp**$(F, A) = \{x \in A : F(x) \neq \emptyset\}$.

(ii) (F, A) is called a **non-null** soft set if supp $(F, A) \neq \emptyset$.

(ii) (F, A) is called a **relative null** soft set denoted by $\emptyset_A$ if $F(x) = \emptyset, \forall x \in A$

(iii) (F, A) is called a **relative whole** soft set, denoted by U_A if $F(x) = U, \forall x \in A$.

Definition 6 (Koyuncu and Tanay, 2016). Let (F, A) be a soft set over U. If $F(x) \neq \emptyset$ for all $x \in A$, then (F, A) is called a **non-empty soft set**.

Definition 7 (Ali *et al.*, 2009). Let (F, A) and (G, B) be two soft sets over U. Then the **union** of (F, A) and (G, B), denoted by $(F, A)\ \tilde{\cup}\ (G, B)$ is a soft set defined as $(F, A)\ \tilde{\cup}\ (G, B) = (H, C), where\ C = A \cup B\ and\ \forall x \in C$,

$$H(x) = \begin{cases} F(x), & if\ x \in A - B \\ G(x), & if x \in B - A \\ F(x) \cup\ G(x), & if\ x \in A \cap B \end{cases}$$

Definition 8 (Koyuncu and Tanay, 2016). Let (F, A) and (G, B) be two soft sets over U. Then the **restricted union** of (F, A) and (G, B), denoted by $(\Gamma, A)\ \tilde{\cup}_R\ (G, B)$ is a soft set defined as;

$(F, A)\ \tilde{\cup}_R\ (G, B) = (H, C)$, where $C = A \cap B\ \neq \emptyset$ and $\forall\ x \in C$, $H(x) = F(x) \cup G(x)$.

Definition 9 (Ali *et al.*, 2009). Let (F, A) and (G, B) be two soft sets over U. Then the **extended intersection** of (F, A) and (G, B), denoted by $(F, A)\ \tilde{\cap}_E\ (G, B)$, is a soft set defined as $(F, A)\ \tilde{\cap}_E\ (G, B) = (H, C)$ where $C = A \cup B$ and $\forall\ x \in C$,

$$H(x) = \begin{cases} F(x), & if\ x \in A - B \\ G(x), & if x \in B - A \\ F(x) \cap\ G(x), & if\ x \in A \cap B \end{cases}$$

Definition 10 (Ali *et al.*, 2009). Let (F, A) and (G, B) be two soft sets over U. Then the **restricted intersection** of (F, A) and (G, B) denoted by $(F, A) \Cap (G, B)$, is a soft set defined as $(F, A) \Cap (G, B) = (H, C)$ where $C = A \cap B$ and $\forall\ x \in C$, $H(x) = F(x) \cap G(x)$.

Definition 11 (Maji *et al.*, 2003). Let (F, A) and (G, B) be two soft sets over U. Then the **AND-product** or **AND-intersection** of (F, A) and (G, B) denoted by $(F, A)\tilde{\wedge}\ (G, B)$ is a soft set defined as

$(F, A)\tilde{\wedge}\ (G, B) = (H, C), where\ C = A \times B\ and\ \forall\ (x, y) \in A \times B, H(x, y) = F(x) \cap G(y)$.

Definition 12 (Maji *et al.*, 2003). Let (F, A) and (G, B) be two soft sets over U. Then the **OR-product** or **OR-union** of (F, A) and (G, B), denoted by $(F, A)\tilde{\vee}\ (G, B)$ is a soft set defined as

$(F, A)\tilde{\vee}\ (G, B) = (H, C), where\ C = A \times B\ and\ \forall\ (x, y) \in A \times B, H(x, y) = F(x) \cup G(y)$.

Fuzzy Soft Set

Let U be an initial universe set and E be a set of parameters (which are fuzzy words or sentences involving fuzzy words). Let $P(U)$ denotes the set of all fuzzy subsets of U, and $A \subseteq E$.

Definition 13 (Maji *et al.*, 2001). A pair (F, A) is called a **fuzzy soft set** over U, where F is a mapping given by $F: A \longrightarrow P(U)$. In other words, a fuzzy soft set over U is a parameterized family of fuzzy subsets of the universe U. For $x \in A$, $F(x)$ may be considered as the set of x-elements or as the set of x-approximate elements of the fuzzy soft set (F, A). Therefore, a fuzzy soft set (F, A) over U can be represented by the set of ordered pairs

$$(F, A) = \{(x, F(x)): x \in A, F(x) \in P(U)\}.$$

Example 2. Suppose that $U = \{h_1, h_2, h_3, h_4, h_5\}$ be a universe set and $E = \{a_1, a_2, a_3, a_4\}$ be a set of parameters $A = \{a_1, a_2, a_3\} \subseteq E$, $F(a_1) = \left\{\frac{h_2}{0.8}, \frac{h_4}{0.6}\right\}$, $F(a_2) = U$ and $F(a_3) = \left\{\frac{h_1}{0.3}, \frac{h_3}{0.4}, \frac{h_5}{0.9}\right\}$, then the fuzzy soft set (F, A) is written as

$$(F, A) = \left\{\left(a_1, \left\{\frac{h_2}{0.8}, \frac{h_4}{0.6}\right\}\right), (a_2, U), \left(a_3, \left\{\frac{h_1}{0.3}, \frac{h_3}{0.4}, \frac{h_5}{0.9}\right\}\right)\right\}.$$

Intuitionistic Fuzzy Soft Set

The following definitions are due to (Yin *et al.,* 2012).

Definition 14. Let U be an initial universe set, E a set of parameters, $I(U)$ denotes the set of all intuitionistic fuzzy subsets of U and $A \subseteq E$. Then a pair $(\hat{F}, A)$ is called an **intuitionistic fuzzy soft set** over U, where $\hat{F}$ is a mapping given by $\hat{F} : A \longrightarrow I(U)$.

In general, for every $e \in A$, $\hat{F}(e)$ is an intuitionistic fuzzy set of U and it is called intuitionistic fuzzy value set of parameter e. Obviously, $\hat{F}(e)$ can be written as an intuitionistic fuzzy set such that $\hat{F}(e) = \left\{\langle x, \mu_{\hat{F}(e)}(x), \lambda_{\hat{F}(e)}(x)\rangle : x \in U\right\}$. Where $\mu_{\hat{F}(e)}$ and $\lambda_{\hat{F}(e)}$ are the membership and non-membership functions, respectively. The set of all intuitionistic fuzzy soft sets over U with parameters from E is called an intuitionistic fuzzy soft class and it is denoted by $I\hat{F}(U, E)$.

Definition 15. Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two intuitionistic fuzzy soft sets over U. We say that $(\hat{F}, A)$ is an **intuitionistic fuzzy soft subset** of $(\hat{G}, B)$ and written as $(\hat{F}, A) \,\tilde{\subseteq}\, (\hat{G}, B)$ if,

(i) $A \subseteq B$,
(i) For any $e \in A$, $\hat{F}(e) \subseteq \hat{G}(e)$, that is, for all $x \in U$ and $e \in A, \mu_{\hat{F}(e)}(x) \leq \mu_{\hat{G}(e)}(x)$ and $\lambda_{\hat{F}(e)}(x) \geq \lambda_{\hat{G}(e)}(x)$.

Definition 16. Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two intuitionistic fuzzy soft sets over U. Then $(\hat{F}, A)$ and $(\hat{G}, B)$ are said to be **intuitionistic fuzzy soft equal**, denoted by $\left(\hat{F}, A\right) = (\hat{G}, B)$ if $(\hat{F}, A) \,\tilde{\subseteq}\, (\hat{G}, B)$ and $\left(\hat{G}, B\right) \,\tilde{\subseteq}\, (\hat{F}, A)$.

Definition 17. Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two intuitionistic fuzzy soft sets over U. Then, the **Union** of $(\hat{F}, A)$ and $(\hat{G}, B)$ is written as $(\hat{F}, A) \,\tilde{\cup}\, (\hat{G}, B)$ and is defined as $(\hat{F}, A) \,\tilde{\cup}\, \left(\hat{G}, B\right) = (\hat{H}, C)$, where $C = A \cup B$ and $\forall\, e \in C$,

$$\hat{H}(e) = \begin{cases} \hat{F}(e), & if\ e \in A/B \\ \hat{G}(e), & if\ e \in B/A\ . \\ \hat{F}(e) \cup \hat{G}(e), & if\ e \in A \cap B \end{cases}$$

Definition 18. Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two intuitionistic fuzzy soft sets over U, such that $A \cap B \neq \emptyset$. The **restricted intersection** of $(\hat{F}, A)$ and $(\hat{G}, B)$ is defined to be the intuitionistic fuzzy soft set $(\hat{H}, C)$, where $C = A \cap B$ and $\hat{H}(e) = \hat{F}(e) \cap \hat{G}(e), \forall e \in C$. This is written as $(\hat{F}, A) \Cap \left(\hat{G}, B\right) = (\hat{H}, C)$.

Definition 19. Let U be an initial universe set, E be the universe set of parameters and $A \subset E$. The intuitionistic fuzzy soft set $(\hat{F}, A)$ is called a **relative null intuitionistic fuzzy soft set** with respect to the parameter set A denoted by $\emptyset_A$, if $\hat{F}(e) =$ null intuitionistic fuzzy set of U, for all $e \in A$. The relative null intuitionistic fuzzy soft set $\emptyset_E$ with respect to the universe set of parameters E is called the **absolute null intuitionistic fuzzy soft set** over U.

Definition 20. Let U be an initial universe set, E be a universe set of parameters and $A \subseteq E$. The intuitionistic fuzzy soft set $(\hat{F}, A)$ is called a **relative whole intuitionistic fuzzy soft set** with respect to the parameter set A denoted by U_A, if $\hat{F}(e) = U$, for all $e \in A$. The relative whole intuitionistic fuzzy soft set U_E with respect to the universe set of parameters E is called the **absolute intuitionistic fuzzy soft set** over U.

Definition 21. The r**elative complement** of an intuitionistic fuzzy soft set $(\hat{F}, A)$ over U is denoted by $(\hat{F}, A)^r$ and is defined by $(\hat{F}^r, A)$, where $\forall e \in A, \mu_{\hat{F}^r(e)} = \lambda_{\hat{F}(e)}$ and $\lambda_{\hat{F}^r(e)} = \mu_{\hat{F}(e)}$, that is, $\hat{F}^r(e) = (\lambda_{\hat{F}(e)}, \mu_{\hat{F}(e)})$. Clearly, $((\hat{F}, A)^r)^r = (\hat{F}, A)$.

Definition 22. Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U, such that $A \cap B \neq \emptyset$. The **restricted difference of** $(\hat{F}, A)$ and $(\hat{G}, B)$ is denoted by $(\hat{F}, A) \sim_R (\hat{G}, B)$ and is defined as $(\hat{F}, A) \sim_R \left(\hat{G}, B\right) = (\hat{K}, P)$, where $P = A \cap B$ and $\forall\, p \in P, \hat{K}(p) = \hat{F}(p) - \hat{G}(p)$ (the intuitionistic fuzzy difference of two intuitionistic fuzzy sets $\hat{F}(p)$ and $\hat{G}(p)$ is denoted by $\hat{F}(p) - \hat{G}(p)$ and is defined as $\hat{F}(p) - \hat{G}(p) = \hat{F}(p) \cap \hat{G}^c(p)$.

Definition 23. Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two intuitionistic fuzzy soft sets over U.The **extended intersection of** $(\hat{F}, A)$ and $(\hat{G}, B)$ denoted by $(\hat{F}, A) \tilde{\cap}_E (\hat{G}, B)$ is defined as

$\left(\hat{F}, A\right) \tilde{\cap}_E \left(\hat{G}, B\right) = (\hat{H}, C)$, $C = A \cup B$, and $\forall e \in C$, $\hat{H}(e) =$

$$\begin{cases} \hat{F}(e), & if\ e \in A/B \\ \hat{G}(e), & if\ e \in B/A\ . \\ \hat{F}(e) \cap \hat{G}(e), & if\ e \in A \cap B \end{cases}$$

Definition 24. Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two intuitionistic fuzzy soft sets over U, such that $A \cap B \neq \emptyset$. The **restricted union** of $(\hat{F}, A)$ and $(\hat{G}, B)$ denoted by $(\hat{F}, A) \tilde{\cup}_R (\hat{G}, B)$ is defined by $\left(\hat{F}, A\right) \tilde{\cup}_R \left(\hat{G}, B\right) = (\hat{H}, C)$ where $C = A \cap B$ and $\hat{H}(e) = \hat{F}(e) \cup \hat{G}(e), \forall e \in C$.

Weighted intuitionistic Fuzzy Soft Set

Definition 25. A **weighted intuitionistic fuzzy soft set** is a triple $\Omega = \langle \hat{F}, A, w \rangle$, where $\langle \hat{F}, A \rangle$ is an intuitionistic fuzzy soft set over the universe set U and $w: A \longrightarrow [0, 1]$ is a weight function specifying the weight $w_j = w(a_j)$, for each parameter $a_j \in A$.

By definition, every intuitionistic fuzzy soft set can be seen or considered as a weighted intuitionistic fuzzy soft set. Obviously, the notion of weighted intuitionistic fuzzy soft set provides a mathematical tool for modeling and analyzing the decision making problems in which all the choice parameters may not be of equal importance. The differences between the significance of attributes are characterized by the weight function in a weighted intuitionistic fuzzy soft set.

Example 3. Let us consider the weighted intuitionistic fuzzy soft set $\boldsymbol{\Omega} = \langle \hat{F},\ A, w \rangle$ which describes the conditions of some states in a country that an investor Mr. X with a budget is considering to site his manufacturing industry.

Suppose that there are six states in the initial universe $U = \{S_1, S_2, S_3, S_4, S_5, S_6\}$ under consideration and that $A = \{a_{1,0.5}, a_{2,0.6}, a_{3,0.7}, a_{4,0.9}, a_{5,0.5}\}$ is a set of decision parameters or attribute related to U with their respective weights attached. The $a_i (i = 1,2,3,4,5)$ stand for the parameters "peaceful", "power supply", "accessible", "densely populated" and "good weather", respectively. Suppose that

$\hat{F}(a_{1,0.5}) =$
$\{\langle S_1, 0.8, 0.2\rangle, \langle S_2, 0.6, 0.2\rangle, \langle S_3, 0.7, 0.2\rangle, \langle S_4, 0.4, 0.3\rangle, \langle S_5, 0.9, 0.1\rangle, \langle S_6, 0.6, 0.4\rangle\}$,

$\hat{F}(a_{2,0.6}) =$
$\{\langle S_1, 0.7, 0.2\rangle, \langle S_2, 0.8, 0.1\rangle, \langle S_3, 0.5, 0.3\rangle, \langle S_4, 0.6, 0.4\rangle, \langle S_5, 0.8, 0.2\rangle, \langle S_6, 0.4, 0.3\rangle\}$,

$\hat{F}(a_{3,0.7}) =$
$\{\langle S_1, 0.7, 0.1\rangle, \langle S_2, 0.6, 0.2\rangle, \langle S_3, 0.3, 0.4\rangle, \langle S_4, 0.8, 0.1\rangle, \langle S_5, 0.9, 0.1\rangle, \langle S_6, 0.6, 0.3\rangle\}$,

$\hat{F}(a_{4,0.9}) =$
$\{\langle S_1, 0.6, 0.3\rangle, \langle S_2, 0.8, 0.1\rangle, \langle S_3, 0.9, 0.1\rangle, \langle S_4, 0.8, 0.2\rangle, \langle S_5, 0.7, 0.1\rangle, \langle S_6, 0.8, 0.2\rangle\}$,

$\hat{F}(a_{5,0.5}) =$
$\{\langle S_1, 0.8, 0.2\rangle, \langle S_2, 0.4, 0.5\rangle, \langle S_3, 0.9, 0.1\rangle, \langle S_4, 0.8, 0.1\rangle, \langle S_5, 0.5, 0.4\rangle, \langle S_6, 0.8, 0.2\rangle\}$.

The weighted intuitionistic fuzzy soft set $\boldsymbol{\Omega} = \langle \hat{F}, A, w\rangle$ is a parameterized family $\{\hat{F}(a_i), i = 1, 2,3,4,5\}$ of fuzzy sets on U and

$$\langle \hat{F}, A, w\rangle =$$

$$\left\{\begin{array}{c} Peaceful\ states = \left\{\begin{array}{c}\langle S_1, 0.8, 0.2\rangle, \langle S_2, 0.6, 0.2\rangle, \langle S_3, 0.7, 0.2\rangle, \langle S_4, 0.4, 0.3\rangle, \langle S_5, 0.9, 0.1\rangle, \\ \langle S_6, 0.6, 0.4\rangle\end{array}\right\}, \\ power\ supply\ states = \left\{\begin{array}{c}\langle S_1, 0.7, 0.2\rangle, \langle S_2, 0.8, 0.1\rangle, \langle S_3, 0.5, 0.3\rangle, \langle S_4, 0.6, 0.4\rangle, \\ \langle S_5, 0.8, 0.2\rangle, \langle S_6, 0.4, 0.3\rangle\end{array}\right\}, \\ accessible\ states = \left\{\begin{array}{c}\langle S_1, 0.7, 0.1\rangle, \langle S_2, 0.6, 0.2\rangle, \langle S_3, 0.3, 0.4\rangle, \langle S_4, 0.8, 0.1\rangle, \\ \langle S_5, 0.9, 0.1\rangle, \langle S_6, 0.6, 0.3\rangle\end{array}\right\}, \\ densely\ populated\ states = \left\{\begin{array}{c}\langle S_1, 0.6, 0.3\rangle, \langle S_2, 0.8, 0.1\rangle, \langle S_3, 0.9, 0.1\rangle, \langle S_4, 0.8, 0.2\rangle, \\ \langle S_5, 0.7, 0.1\rangle, \langle S_6, 0.8, 0.2\rangle\end{array}\right\}, \\ states\ with\ good\ weather = \left\{\begin{array}{c}\langle S_1, 0.8, 0.2\rangle, \langle S_2, 0.4, 0.5\rangle, \langle S_3, 0.9, 0.1\rangle, \langle S_4, 0.8, 0.1\rangle, \\ \langle S_5, 0.5, 0.4\rangle, \langle S_6, 0.8, 0.2\rangle\end{array}\right\} \end{array}\right\}$$

Table 1 gives the tabular representation of weighted intuitionistic fuzzy soft set in Example 3.

Table 1: Tabular representation of weighted intuitionistic fuzzy soft set $\Omega = \langle \hat{F}, A, w\rangle$

U/A	$a_{1,0.5}$	$a_{2,0.6}$	$a_{3,0.7}$	$a_{4,0.9}$	$a_{5,}$
S_1	$\langle 0.8, 0.2\rangle$	$\langle 0.7, 0.2\rangle$	$\langle 0.7, 0.1\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.8, 0.2\rangle$
S_2	$\langle 0.6, 0.2\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.6, 0.2\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.4, 0.5\rangle$
S_3	$\langle 0.7, 0.2\rangle$	$\langle 0.5, 0.3\rangle$	$\langle 0.3, 0.4\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.9, 0.1\rangle$
S_4	$\langle 0.4, 0.3\rangle$	$\langle 0.6, 0.4\rangle$	$\langle 0.8, 0.1\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.8, 0.1\rangle$
S_5	$\langle 0.9, 0.1\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.9, 0.1\rangle$	$\langle 0.7, 0.1\rangle$	$\langle 0.5, 0.4\rangle$
S_6	$\langle 0.6, 0.4\rangle$	$\langle 0.4, 0.3\rangle$	$\langle 0.6, 0.3\rangle$	$\langle 0.8, 0.2\rangle$	$\langle 0.8, 0.2\rangle$

Weighted Intuitionistic Fuzzy Soft Set Based Decision Making

Level soft set of a weighted intuitionistic fuzzy soft set

Lin in 1996 defined a theory of mathematical analysis, namely the weighted soft set (W-soft sets). In line with Lin's style, Maji et al. (2002) defined the weighted table of soft set. A weighted table of soft set is presented by $d_{ij} = w_j \times h_{ij}$ instead of 0 and 1 only, where h_{ij} are entries in the table of the soft set and w_j are the weights of the attributes e_j. The weighted choice value of an object o_i is c_i, given by $c_i = \sum_j d_{ij}$.

Definition 26. Let $\Omega = \langle \hat{F}, A, w\rangle$ be a weighted intuitionistic fuzzy soft set, where $\langle \hat{F},\ A\rangle$ is an intuitionistic fuzzy soft set over U and A is a set of parameters. For $s, t \in [0, 1]$, the $(\boldsymbol{s}, \boldsymbol{t}) - \boldsymbol{level}$ **soft set** of Ω is a standard soft set $L(\Omega; s, t) = \langle \hat{F}_{(s,t)}, A_W\rangle$ such that for every $a \in A\ \exists\ w \in [0, 1]$ defined by $\hat{F}_{(s,t)}(a_w) = L(\hat{F}(a_w); s, t) = \{x \in U: \mu_{\hat{F}(a_w)}(x) \geq s \text{ and } \lambda_{\hat{F}(a_w)}(x) \leq t\}$.

This definition is clearly an extension of level soft sets of fuzzy soft sets. That is, $s \in [0, 1]$ can be seen as a given **least threshold** on membership values and $t \in [0, 1]$ can be seen as a given **greatest threshold** on non-membership values. In a real-life application of weighted intuitionistic fuzzy soft set based decision making, normally the thresholds are chosen in advance by the decision maker(s) and represent their requirements on membership levels and non-membership levels, respectively. While $w \in [0, 1]$ is a fixed value imposed on each parameter by the decision maker(s).

Definition 27. Let $\Omega = \langle \hat{F}, A, w\rangle$ be a weighted intuitionistic fuzzy soft set, where $\langle \hat{F},\ A\rangle$ is an intuitionistic fuzzy soft set over U and A is a set of parameters. Let $\eta: A \to [0, 1] \times [0, 1]$ be an intuitionistic fuzzy set in A which is called a **Threshold intuitionistic fuzzy set.** The level soft set of Ω with respect to η is a crisp soft set $L(\Omega, \eta) = \langle \hat{F}_{\Omega}, A_w\rangle$ such that $\forall\ a \in A\ \exists\ w \in [0, 1]$ defined by

$$\hat{F}_{\eta}(a_w) = L\left(\hat{F}(a_w);\ \eta(a_w)\right) = \{x \in U: \mu_{\hat{F}(a_w)}(x) \geq\ \mu_{\eta}(a_w) \text{ and } \lambda_{\hat{F}(a_w)}(x) \leq \lambda_{\eta}(a_w)\}.$$

Clearly, the level soft sets of weighted intuitionistic fuzzy soft sets with respect to an intuitionistic fuzzy set are extensions of the level soft set.

Definition 28. (**The mid-level soft set** of a weighted intuitionistic fuzzy soft set). Let $\boldsymbol{\Omega} = \langle \hat{F},\ A, w\rangle$ be a weighted intuitionistic fuzzy soft set, where $\langle \hat{F},\ A\rangle$ is an intuitionistic fuzzy soft set over U and A is a set of parameters. Based on the weighted intuitionistic fuzzy soft set $\boldsymbol{\Omega} = \langle \hat{F},\ A, w\rangle$**,** we can define an intuitionistic fuzzy set

$mid_{\boldsymbol{\Omega}}: A \rightarrow [0,1] \times [0,1]$, such that $\forall\, a \in A\ \exists\, w \in [0,1]$ defined by $\mu_{mid\boldsymbol{\Omega}}(a_w) = \frac{1}{|U|}\sum_{x\in U} \mu_{\hat{F}(a_w)}(x)$ and $\lambda_{mid\boldsymbol{\Omega}}(a_w) = \frac{1}{|U|}\sum_{x\in U} \lambda_{\hat{F}(a_w)}(x)$. The intuitionistic fuzzy set $mid_{\boldsymbol{\Omega}}$ is called the **mid-threshold** of the weighted intuitionistic fuzzy soft set $\boldsymbol{\Omega}$**.** In addition, the level soft set of $\boldsymbol{\Omega}$ with respect to the mid-threshold intuitionistic fuzzy set $mid_{\boldsymbol{\Omega}}$, namely $L(\boldsymbol{\Omega}, mid_{\boldsymbol{\Omega}})$ is called the mid-level soft set of $\boldsymbol{\Omega}$ and is represented simply by $L(\boldsymbol{\Omega}; mid)$. In what follows the mid-level decision rule will mean using the mid-threshold and considering the mid-level soft set in weighted intuitionistic fuzzy soft sets based decision making.

Definition 29. (The **Top-Top-level soft set** and **Bottom-Bottom-level soft set** of a weighted intuitionistic fuzzy soft set). Let $\boldsymbol{\Omega} = \langle \hat{F},\ A, w\rangle$ be a weighted intuitionistic fuzzy soft set, where $\langle \hat{F},\ A\rangle$ is an intuitionistic fuzzy soft set over U and A is a set of parameters. Based on the intuitionistic fuzzy soft set

$\boldsymbol{\Omega} = \langle \hat{F}, A, w\rangle$**,**

Intuitionistic fuzzy set $\boldsymbol{toptop_{\Omega}}: A \rightarrow [0,1] \times [0,1]$ such that $\forall\, a \in A\ \exists\, w \in [0,1]$ defined by

$\mu_{toptop\,\boldsymbol{\Omega}}(a_w) = \max_{x\in U} \mu_{\hat{F}(a_w)}(x)$ and $\lambda_{toptop\,\boldsymbol{\Omega}}(a_w) = \max_{x\in U} \lambda_{\hat{F}(a_w)}(x)$.

Also, intuitionistic fuzzy set $\boldsymbol{bottombottom_{\Omega}}: A \rightarrow [0,1] \times [0,1]$ such that $\forall\, a \in A\ \exists\, w \in [0,1]$ define by

$\mu_{bottombottom\,\boldsymbol{\Omega}}(a_w) = \min_{x\in U} \mu_{\hat{F}(a_w)}(x)$ and $\lambda_{bottombottom\,\boldsymbol{\Omega}}(a_w) = \min_{x\in U} \lambda_{\hat{F}(a_w)}$.

To illustrate the above definitions, we shall consider the following Example 3.

The algorithm below itemized the steps to be taken in arriving at an accurate decision,

Algorithm

(I) Input the weighted intuitionistic fuzzy soft set $\boldsymbol{\Omega} = \langle \hat{F}, A, w\rangle$**.**

(II) Input a threshold intuitionistic fuzzy set $\eta: A \rightarrow [0,1] \times [0,1]$ (or give a threshold value pair $(s,t) \in [0,1] \times [0,1]$; or choose a mid-level decision rule; or choose the top-top-level decision rule; or choose the bottom-bottom-level decision rule) for decision making.

(III) Compute the level soft set $L(\boldsymbol{\Omega}; \eta)$ with respect to the threshold intuitionistic fuzzy set η (or the (s,t) −level soft set $L(\boldsymbol{\Omega}; s, t)$; or the mid-level soft set $L(\boldsymbol{\Omega}; mid)$; or the top-top-level soft set $L(\boldsymbol{\Omega}; toptop)$ or the bottom-bottom-level soft set $L(\boldsymbol{\Omega}; bottombottom)$).

(IV) Present the level soft set $L(\boldsymbol{\Omega}; \eta)$ (or $L(\boldsymbol{\Omega}; s, t)$; $L(\boldsymbol{\Omega}; mid); or\ L(\boldsymbol{\Omega}; topbottom)$; $orL(\boldsymbol{\Omega}; toptop); or\ L(\boldsymbol{\Omega}; bottombottom)$) in tabular form and compute the choice value c_i of o_i, for all i.

(i) The optimal decision is to select o_k if $c_k = max_i c_i$.

(ii) If k has more than one value then any one of o_k may be chosen.

We now consider $(\boldsymbol{s},\boldsymbol{t}) - \boldsymbol{level}$ soft set of $\boldsymbol{\Omega}$ using Example 3.

Let us take $S = 0.7$ and $t = 0.3$, then we have the following:

$L(\hat{F}(a_{1,0.5}); 0.7, 0.3) = \{S_1, S_3, S_5\}$,

$L(\hat{F}(a_{2,0.6}); 0.7, 0.3) = \{S_1, S_2, S_5\}$,

$L(\hat{F}(a_{3,0.7}); 0.7, 0.3) = \{S_1, S_4, S_5\}$,

$L(\hat{F}(a_{4,0.9}); 0.7, 0.3) = \{S_2, S_3, S_4, S_5, S_6\}$,

$L(\hat{F}(a_{5,0.5}); 0.7, 0.3) = \{S_1, S_3, S_4, S_6\}$.

Hence, the $(0.7, 0.3)$ −level soft set $\boldsymbol{\Omega} = \langle \hat{F}, A, w\rangle$ is a soft set $L(\boldsymbol{\Omega}; 0.7, 0.3) = \langle \hat{F}_{(0.7,0.3)}, A\rangle$, where the set- valued mapping $\hat{F}_{(0.7,0.3)}: A \rightarrow P(U)$ is defined by

$\hat{F}_{(0.7,0.3)}(a_{i,w}) = L(\hat{F}(a_{i,w}); 0.7,0.3)$ for $i = 1,2,3,4,5$. Table 2 gives the tabular representation of the $(0.7,0.3) - level$ soft set $L(\boldsymbol{\Omega}; 0.7, 0.3)$ with choice value.

Table 2: Tabular representation of the level soft set $L(\boldsymbol{\Omega}; \mathbf{0.7}, \mathbf{0.3})$ with choice value

U/A	$a_{1,0.5}$	$a_{2,0.6}$	$a_{3,0.7}$	$a_{4,0.9}$	$a_{5,0.5}$	$Choice\ Value\ (c_i)$
S_1 .	1	1	1	0	1	2.3
S_2 .	0	1	0	1	0	1.5
S_3 .	1	0	0	1	1	1.9
S_4 .	0	0	1	1	1	2.1
S_5 .	1	1	1	1	0	2.7
S_6 .	0	0	0	1	1	1.4

From the Table **2,** it follows that, the maximum choice value is $c_5 = \mathbf{2.7}$ and therefore the optimal decision is to select state S_5.

For mid-level soft sets, let us consider Example **3** and $\boldsymbol{\Omega} = \langle \hat{F}, A, w\rangle$ with tabular representation in Table 3**.**

it is clear that, the mid-threshold of $\langle \hat{F}, A, w\rangle$ is an intuitionistic fuzzy set

$mid_{\langle \hat{F},A\rangle} = \{\langle a_1, 0.67, 0.23\rangle, \langle a_2, 0.63, 0.25\rangle, \langle a_3, 0.65, 0.2\rangle, \langle a_4, 0.77, 0.16\rangle, \langle a_5, 0.7, 0.25\rangle\}$, and the mid-level soft set of $\langle \hat{F}, A\rangle$ is a soft set $L(\langle \hat{F}, A\rangle; mid)$ with its tabular representation given by **Table 3.**

Table 3: Tabular representation of the mid-level soft set $L(\langle \hat{F}, A\rangle; mid)$ with choice value

U/A	$a_{1,0.5}$	$a_{2,0.6}$	$a_{3,0.7}$	$a_{4,0.9}$	$a_{5,0.5}$	$Choice\ Value\ (c_i)$
S_1.	1	1	1	0	1	2.3
S_2.	0	1	0	1	0	1.5
S_3.	1	0	0	1	1	1.9
S_4.	0	0	1	0	1	1.2
S_5.	1	1	1	0	0	1.8
S_6.	0	0	0	0	1	0.5

From the Table 3, the maximum choice value is $c_1 = 2.3$ and so the optimal decision is to select state S_1.

For illustrative example of **top-top-level soft sets** and **bottom-bottom-level soft sets**, let us again consider the weighted intuitionistic fuzzy soft set $\boldsymbol{\Omega} = \langle \hat{F}, A, w\rangle$ with its tabular representation given by Table 4.

it is obvious that the top-top-threshold of $\langle \hat{F}, A, w\rangle$ is an intuitionistic fuzzy set

$toptop_{\langle \hat{F},A\rangle} = \{\langle a_1, 0.9, 0.4\rangle, \langle a_2, 0.8, 0.4\rangle, \langle a_3, 0.9, 0.4\rangle, \langle a_4, 0.9, 0.3\rangle, \langle a_5, 0.9, 0.5\rangle\}$. and the top-top-level soft set of $\langle \hat{F}, A\rangle$ is a soft set $L(\langle \hat{F}, A\rangle; toptop)$ with its tabular representation given by Table 4.

It is clear that the bottom-bottom-threshold of $\langle \hat{F}, A, w\rangle$ is an intuitionistic fuzzy set

$bottombottom_{\langle \hat{F},A\rangle} = \{\langle a_1, 0.4, 0.1\rangle, \langle a_2, 0.4, 0.1\rangle, \langle a_3, 0.3, 0.1\rangle, \langle a_4, 0.6, 0.1\rangle, \langle a_5, 0.4, 0.1\rangle\}$, and the bottom-bottom-level soft set of $\langle \hat{F}, A\rangle$ is a soft set $L(\langle \hat{F}, A\rangle; bottombottom)$ with its tabular representation given by Table 4.

Table 4: Tabular representation of the top-top-level soft set $L(\Omega; toptop)$ with choice value

U/A	$a_{1,0.5}$	$a_{2,0.6}$	$a_{3,0.7}$	$a_{4,0.9}$	$a_{5,0.5}$	$Choice\ Value\ (c_i)$
S_1.	0	0	0	0	0	0.0
S_2.	0	1	0	0	0	0.6
S_3.	0	0	0	1	1	1.4
S_4.	0	0	0	0	0	0.0
S_5.	1	1	1	0	0	1.8
S_6.	0	0	0	0	0	0.0

From Table 4, it follows that the maximum choice value is $c_5 = \mathbf{1.8}$**.** The optimum decision is to select state S_5**.**

Table 5: Tabular representation of the bottom-bottom-level soft set

$L(\Omega; bottombottom)$ with choice value

U/A	$a_{1,0.5}$	$a_{2,0.6}$	$a_{3,0.7}$	$a_{4,0.9}$	$a_{5,0.5}$	$Choice\ Value\ (c_i)$
S_1.	0	0	1	0	0	0.7
S_2.	0	1	0	1	0	1.5
S_3.	0	0	0	1	1	1.4
S_4.	0	0	1	0	1	1.2
S_5.	1	0	1	1	0	1.6
S_6.	0	0	0	0	0	0.0

From Table 5, it follows that the maximum choice value is $c_5 = \mathbf{1.6}$**.** The optimum decision is to select state S_5**.**

Also, after employing all decision rules and imposing weights on the parameters using Example 3, we observed that the choice of the states differs and this is due to the decision maker's preference (or the choice of the decision rules).

Conclusion

In this paper, we introduced the concept of weighted intuitionistic fuzzy soft set which enhance the study of hybrid soft sets. Finally, an adjustable approach to decision making problem using level soft set of a weighted intuitionistic fuzzy soft set was presented with illustrative examples. Weighted intuitionistic fuzzy soft set enables a decision maker(s) to arrive at an accurate decision as illustrated in example 3.

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